Ch4. Comparative Statics and Demand

EX： $u(x,y) = 2\sqrt{x} + \sqrt{y}$ ， $Max_{x,y}\ 2\sqrt{x} + \sqrt{y}\quad s.t.\ P_x\,x + P_y\,y = m$

$$FOC \Rightarrow MRS_{xy} = \frac{P_x}{P_y}$$

$$P_x\,x + P_y\,y = m$$

$$MRS_{xy} = \frac{MU_x}{MU_y} = \frac{2*0.5*x^{-0.5}}{1} = \frac{P_x}{P_y}$$

$$\frac{1}{x^{0.5}} = \frac{P_x}{P_y}$$

$$x^* = \frac{{P_y}^2}{{P_x}^2} \quad \text{------ ①}$$

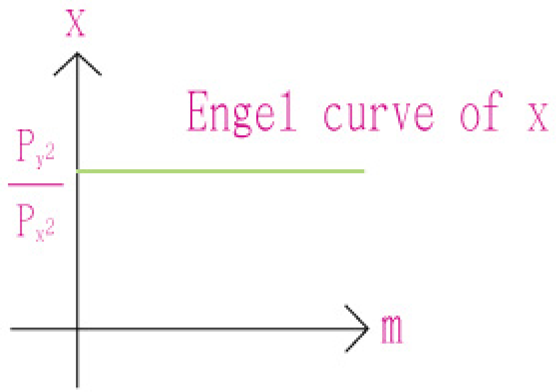
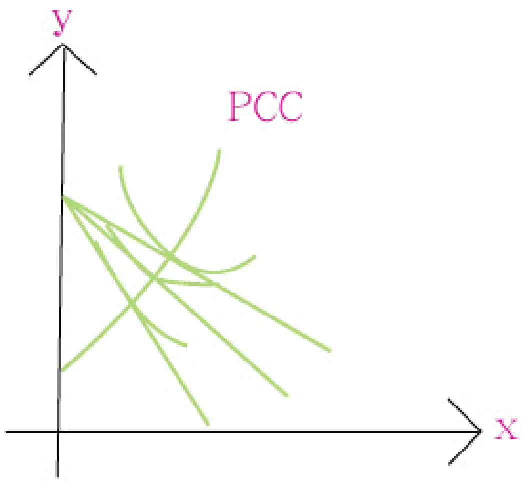


Figure44 :Engel curve

$$\frac{{P_y}^2}{P_x} + P_y\,y = m$$

$$P_y\,y = m - \frac{{P_y}^2}{P_x} \qquad y^* = \frac{m}{P_y} - \frac{P_y}{P_x} \quad \text{------ ②}$$

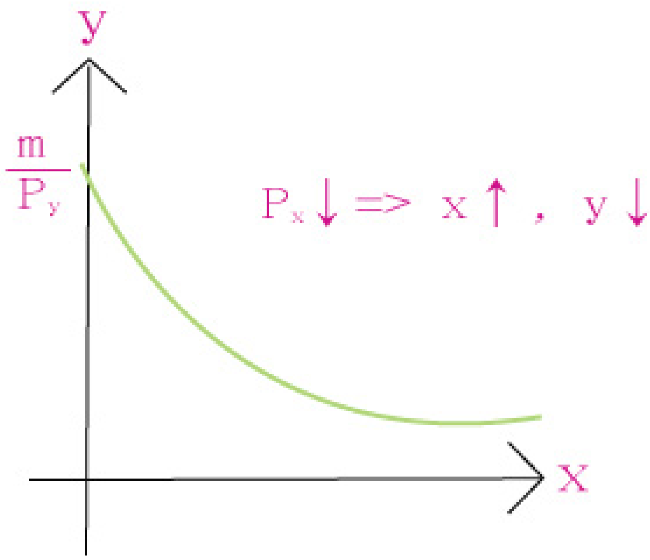
① → ② $y = \frac{m}{P_y} - x^{0.5}$



$\frac{d_y}{d_x} = -0.5\, x^{-0.5}$

$\frac{d^2{}_y}{{d_x}^2} = 0.25\, x^{-1.5} \ > 0$

Figure45 :Price consumer curve



not a Giffen good

X & *Y* are substitutes

From the demand curves.

Figure46 : Demand curve that X&*Y* substitutes

$x^* = \frac{{P_y}^2}{{P_x}^2} \Rightarrow \quad P_x \uparrow\ ,\ x^* \downarrow$

$\&\quad P_y \uparrow\ ,\ x^* \uparrow$

x^* independent of income.

$y^* = \frac{m}{P_y} - \frac{P_y}{P_x} \Rightarrow\ P_x \uparrow\ ,\ y^* \uparrow\ \ \&\ P_y \uparrow, y^* \downarrow\ \&\ m \uparrow\ ,\ y^* \uparrow$ $\quad$ *Y* is a normal good.

Market demand = Sum of the individual demand

EX： Inverse demand function： Demand curve

consumer A： $P_x = 10 - 2X_A$, $X_A = 5 - 0.5P_x$

B： $P_x = 5 - 3X_B$, $X_B = \frac{5}{3} - \frac{1}{3}P_x$

C： $P_x = 12 - 6X_C$, $X_{CB} = 2 - \frac{1}{6}P_x$

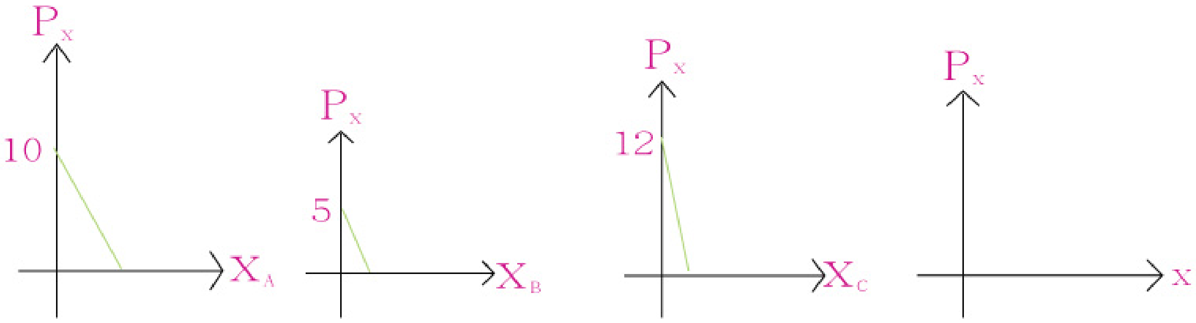


Figure47 :**Horizontal aggregation in Demand curve**

$12 \le P_x \Rightarrow X_A = 0,\quad X_B = 0,\quad X_C = 0,\quad x = X_A + X_B + X_C = 0$

$10 \le P_x \le 1 \Rightarrow X_A = 0,\quad X_B = 0,\quad X_C = 2 - \frac{1}{6}P_x,\quad x = X_C = 2 - \frac{1}{6}P_x$

$5 \le P_x \le 10 \Rightarrow X_A = 5 - \frac{1}{2}P_x,\quad X_B = 0,\quad X_C = 2 - \frac{1}{6}P_x,\quad x = X_A + X_C = 7 - \frac{2}{3}P_x$

$0 \le P_x \le 5 \Rightarrow X_A = 5 - \frac{1}{2}P_x,\quad X_B = \frac{5}{3} - \frac{1}{3}P_x,\quad X_C = 2 - \frac{1}{6}P_x,\quad x = \frac{26}{3} - P_x$

***Conclusion：** 1. Market demand "curve" is the "horizontal" sum of the individual demand curves.

2. Individual demand curves are downward sloping.

⇒ market demand curve is downward sloping.

***Property of the market demand curve：**

Downward sloping (usually)

slope represents quantity demanded depends on price.

price ↓ , quantity demanded ↑

⇒ sensitivity of the change of the quantity demanded with the change with the change in price (price of other good, income, ... other determinants)

*Price Elasticity of demand (of good X)

$$\varepsilon_x^p = \left| \frac{\frac{\Delta x}{x}}{\frac{\Delta Px}{Px}} \right| = \left| \frac{\text{percentage change in quantity demanded of X}}{\text{percentage change in price of X}} \right|$$

$$= \left| \frac{\Delta x}{\Delta Px} \frac{Px}{x} \right|$$

slope of the demand function

ε_x^p depends on $\frac{dx}{dPx}$ $, Px, x$

determinants ：

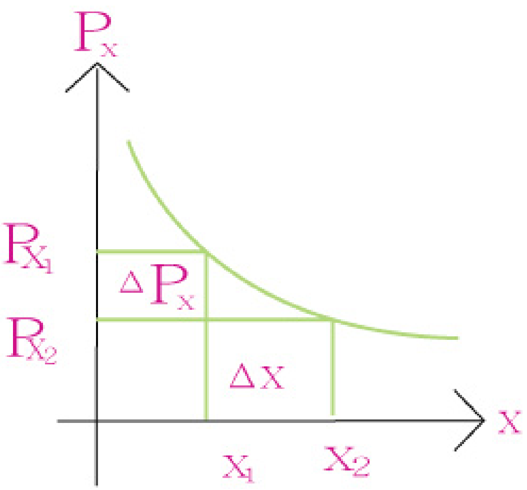
1. property of the good X (necessity ?)
2. substitute availability
3. the proportion of the expenditure of X with respect to the income.
4. time frame.

$\varepsilon_x^p = 1$ unit elastic

$\varepsilon_x^p > 1$ elastic

$\varepsilon_x^p < 1$ inelastic.

if $P_x \to 0$, $\varepsilon_x^p = \left| \frac{\partial x}{\partial P_x} \frac{P_x}{x} \right|$



Ｆigure48： **Elasticity concept (change in *X* response to *P*)**

$(\frac{P_{x2}}{x_2})$

$\varepsilon_x^p = \left|\frac{x_2 - x_1}{P_{x2} - P_{x1}} \frac{P_{x1}}{x_1}\right|$ (point elasticity of demand)

$\varepsilon_x^p = \left|\frac{x_2 - x_1}{P_{x2} - P_{x1}} \frac{\frac{P_{x1}+P_{x2}}{2}}{\frac{x_1+x_2}{2}}\right| = \left|\frac{x_2 - x_1}{P_{x2} - P_{x1}} \frac{\frac{P_{x1}+P_{x2}}{2}}{\frac{x_1+x_2}{2}}\right| = \left|\frac{x_2 - x_1}{P_{x2} - P_{x1}} \frac{P_{x1}+P_x}{x_1+x}\right|$(arc elasticity of demand)

$\varepsilon_x^p = \left|\frac{\frac{\Delta x}{x}}{\frac{\Delta Px}{Px}}\right|$ $\Delta x > 0 \Rightarrow P_x X \uparrow$

$\Delta P_x < 0 \Rightarrow P_x X \downarrow$

P_x changes $\Rightarrow$ Total expenditure ($P_x X$) of X doesn't change if $\varepsilon_x^p = 1$

$P_x \downarrow\uparrow \Rightarrow x \uparrow\downarrow$ and $\varepsilon_x^p > 1$

$\Rightarrow$ expenditure of $X \uparrow\downarrow$

$P_x \downarrow\uparrow \Rightarrow x \uparrow\downarrow$ and $\varepsilon_x^p < 1$

$\Rightarrow$ expenditure of $X \downarrow\uparrow$ fixed

$e_x = P_x x$ $x(P_x, P_y, m), \dots$

$\frac{de_x}{dP_x} = \frac{dP_x x}{dP_x} = \frac{P_x dx}{dP_x} + \frac{x dP_x}{dP_x}$

$= P_x \frac{dx}{dP_x} + x$

$= x\left(\frac{dx}{dP_x}\frac{P_x}{x} + 1\right)$

$= x(1 - \varepsilon_x^p)$

$\frac{de_x}{dP_x} = \frac{dP_x x}{dP_x} < 0 \quad if \ \varepsilon_x^p > 1$ elastic

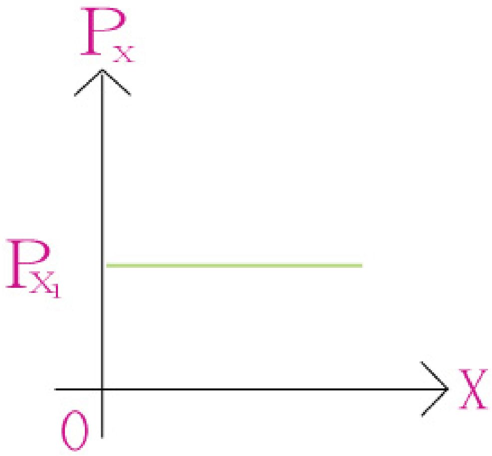
$$\frac{de_x}{dP_x} = \frac{dP_x\,x}{dP_x} > 0 \quad if\ \varepsilon_x^p < 1 \quad \text{inelastic.}$$

$$\frac{de_x}{dP_x} = \frac{dP_x\,x}{dP_x} = 0 \quad if\ \varepsilon_x^p = 1 \quad \text{unit elastic}$$

＊Special case

1.Horizontal demand curve

$\varepsilon_x^p \rightarrow \infty$ at all points on the demand curve.

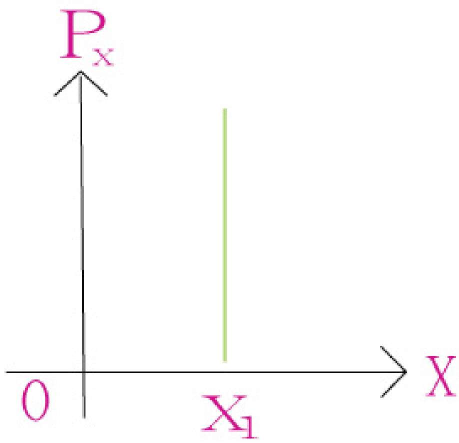


Ｆigure49 **:Perfect elastic Demand**

2.Vertical demand curve

非用不可的商品　ex.癌症治療藥物

$\varepsilon_x^p = 0$



Ｆigure50 **:perfect inelastic Demand**

3.Linear demand curve

$$P_x = a\text{-}bX \qquad \varepsilon_x^p \ \ at\ d = \left|\frac{dx}{dP_x}\ \frac{P_x}{x}\right|$$

$$X = \frac{a}{b} - \frac{P_x}{b} \qquad = \left|\frac{1}{b}\ \frac{a-bX}{x}\right| \quad \text{note that } b = \frac{oa}{oc} = \frac{fa}{fd}$$

$$= \frac{fd}{fa} \cdot \frac{of}{og}$$

$$= \frac{\cancel{og}}{fa} \cdot \frac{of}{\cancel{og}}$$

$$= \frac{dc}{da}$$

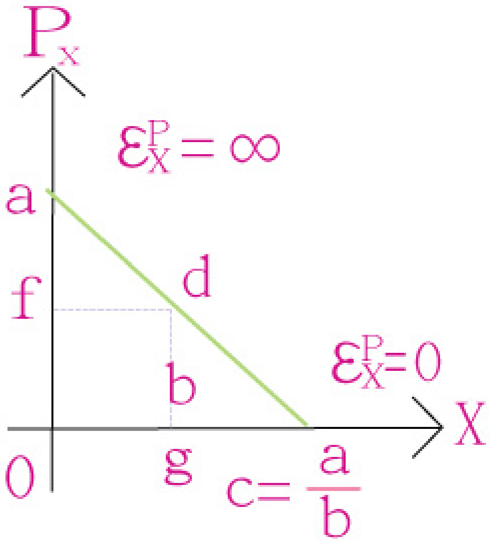


Figure51:Elasticity of linear Demand curve

⇨ $\varepsilon_x^p = 1$ if $dc = da$

i.e. d is exactly the middle point of the demand curve.

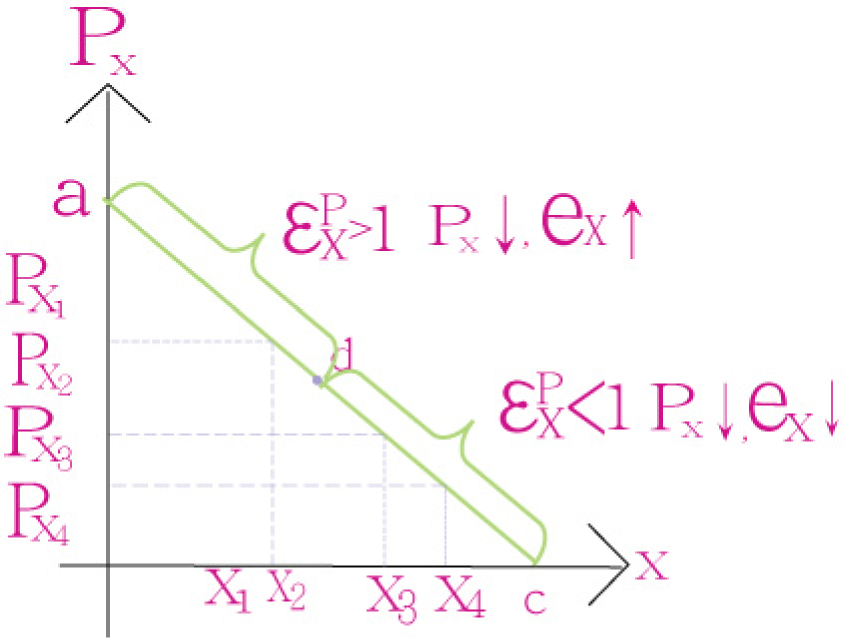


Figure52：Elasicity of linear demand curve

$$\varepsilon_x^p \;\; at\ d = \left| \frac{dx}{dP_x} \frac{P_x}{x} \right|$$

$$= \left| \frac{1}{\frac{af}{fd}} \cdot \frac{of}{og} \right|$$

$$= \frac{\mathrm{fd}}{\mathrm{af}} \cdot \frac{\mathrm{of}}{\mathrm{og}}$$

$$= \frac{\sout{\mathrm{og}}}{\mathrm{af}} \cdot \frac{\mathrm{of}}{\sout{\mathrm{og}}}$$

$$= \frac{\mathrm{of}}{\mathrm{af}} = \frac{\mathrm{dc}}{\mathrm{da}}$$

$$\varepsilon_x^p \text{ at d} = \frac{d'c'}{d'a'} > \frac{dc}{da}$$

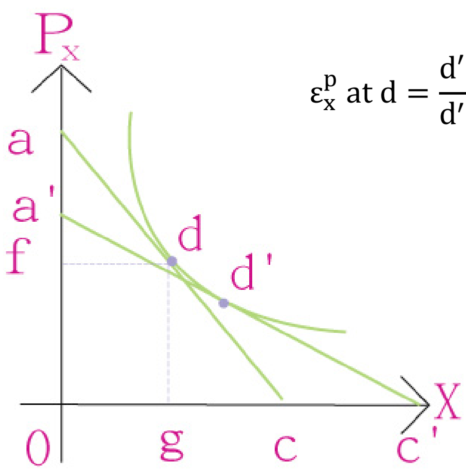


Figure53： Price elasticity at demand

∗ Demand curve with $\varepsilon_x^p = 1$ at all points in the demand curve.

$P_{x1}X_1 = P_{x2}X_2$

$P_xX = \text{constant} = -e$

$$X = \frac{e}{P_x} = eP_x^{-1}$$

$$\varepsilon_x^p = \left|\frac{dx}{dP_x} \cdot \frac{P_x}{x}\right|$$

$$= \left|-\frac{e}{P_{x^2}} \cdot \frac{P_x}{\frac{e}{P_x}}\right|$$

$$= |-1|$$

$$= 1$$

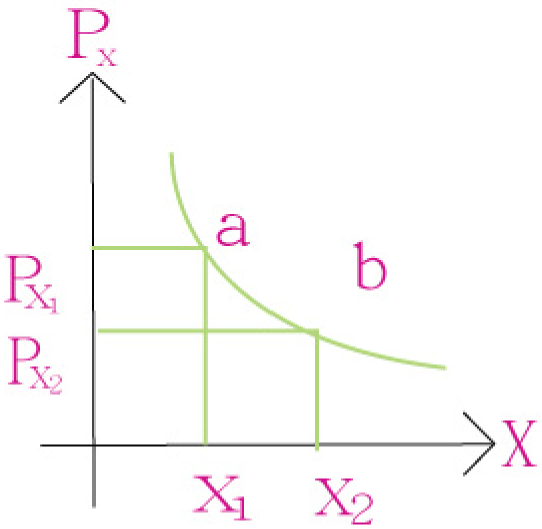


Figure54 :Demand curve

∗ another formula of ε_x^p

$\frac{dlnx}{dx} = \frac{1}{x}$ $\qquad dlnx = \frac{1}{x} \cdot dx$

$\frac{dlnP_x}{dP_x} = \frac{1}{P_x}$ $\qquad dlnP_x = \frac{1}{P_x} \cdot dP_x$

$\frac{dlnx}{dlnP_x} = \frac{\frac{1}{x}dx}{\frac{1}{P_x}dP_x} = \frac{dx}{dP_x}\frac{P_x}{x}$ $\quad => \quad \varepsilon_x^p = \left|\frac{dlnx}{dlnP_x}\right|$

$\left|\frac{dlnx}{dlnP_x}\right| = \varepsilon$

$\frac{dlnx}{dlnP_x} = -\varepsilon$ $\qquad Let \begin{matrix} lnx=s \\ lnP_x=t \end{matrix}$ $\qquad \frac{dlnx}{dlnP_x} = \frac{ds}{dt}$

$$\int \frac{ds}{dt} dt = s = lnx$$

$\varepsilon_x^p = \varepsilon$ some constant at all points on a demand curve.

$x = eP_x{}^{-\varepsilon} = \frac{e}{P_x{}^{\varepsilon}}$ (constant)

$$\varepsilon_x^p = \left|\frac{dlnx}{dlnP_x}\right| = \left|\frac{dlneP_x{}^{-\varepsilon}}{dlnP_x}\right| = \left|\frac{d(lne-\varepsilon lnP_x)}{d(lnP_x)}\right|$$

$$= \left|\frac{-\varepsilon dlnP_x}{dlnP_x}\right| = |-\varepsilon| = \varepsilon$$

Suppose $\varepsilon_x^p = \varepsilon$ Demand function x = ?

$$\varepsilon_x^p = \left|\frac{dlnx}{dlnP_x}\right| = -\frac{dlnx}{dlnP_x}$$

$$-\frac{dlnx}{dlnP_x} = \varepsilon \qquad \frac{dlnx}{dlnP_x} = -\varepsilon$$

$$\int \frac{dlnx}{dlnP_x} dlnP_x = \int -\varepsilon dlnP_x$$

$$lnx = -\varepsilon dlnP_x + C$$

$$x^{lnx} = e^{-\varepsilon dlnP_x + C}$$

$$X = e^{-\varepsilon dlnP_x} e^c = e^c e^{lnP_x{}^{-\varepsilon}} = e^c P_x{}^{-\varepsilon}$$

∗Income elasticity of demand

$$\varepsilon_x^m = \frac{\frac{\Delta x}{x}}{\frac{\Delta m}{m}} = \frac{\Delta x}{\Delta m}\frac{m}{x}$$

$\Delta m \to 0$

$$\varepsilon_x^m = \frac{\partial x}{\partial m}\frac{m}{x}$$

Engel curve given

$X = x(P_x, P_y, m)$ $X = x(m|\overline{P_x, P_y})$

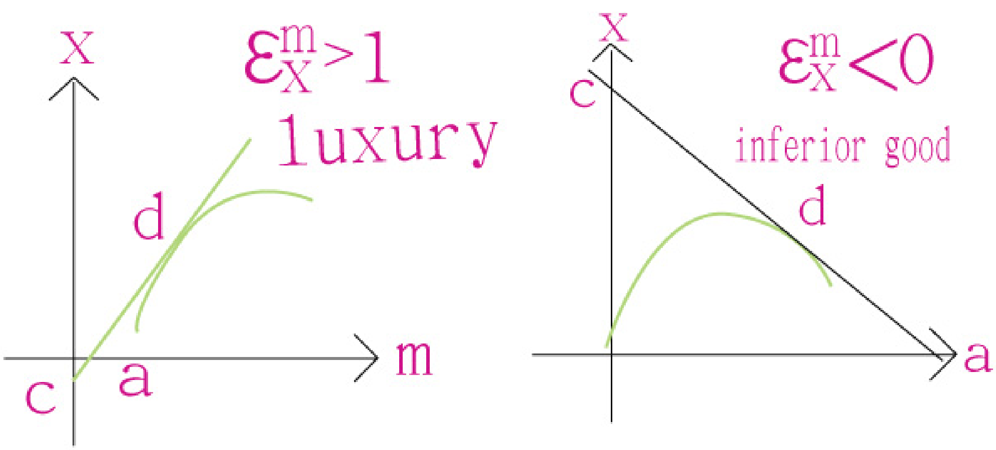


Figure55 :use engel curve to demostrate income elasticity

ε_x^m at d = ?

$$\varepsilon_x^m = \frac{\partial x}{\partial m} \cdot \frac{m}{x} \text{ at } d$$

$$= \frac{fd}{fa} \cdot \frac{of}{og}$$

$$= \frac{\cancel{og}}{fa} \cdot \frac{of}{\cancel{og}} = \frac{fo}{fa} = \frac{dc}{da}$$

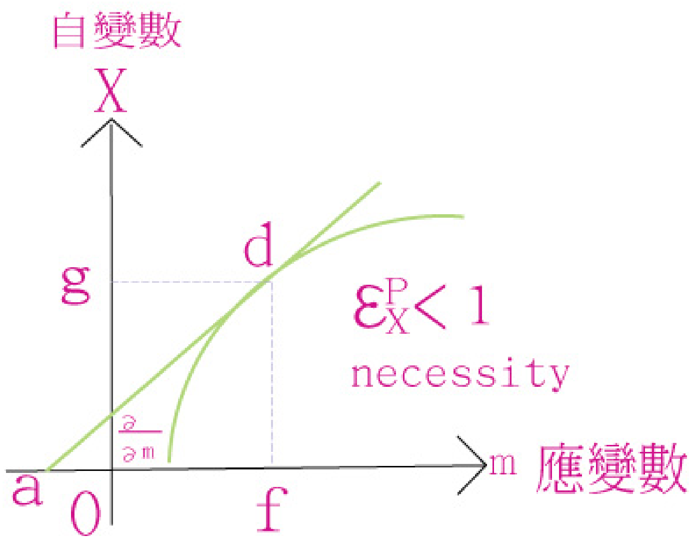


Figure56 :use engel curve to demostrate income elasticity

＊Definition: $\varepsilon_x^m < 0$ => inferioi good $m\uparrow, x^*\downarrow$

$0 < \varepsilon_x^m < 1$ => necessity

$\varepsilon_x^m > 1$ => luxury

＊Cross Elastivity of Demand

$X = x(P_x, P_y, m)$ $\quad \varepsilon_{xy} = \dfrac{\frac{\Delta x}{x}}{\frac{\Delta P_y}{P_y}} = \dfrac{\Delta x}{\Delta P_y}\dfrac{P_y}{x}$

$\Delta P_y \to 0 \quad \varepsilon_{xy} = \dfrac{\partial x}{\partial P_y}\dfrac{P_y}{x}$

$\varepsilon_{xy} > 0$ => X & Y are substitutes.

$\varepsilon_{xy} < 0$ => X & Y are complements.

Budget constraint

$P_x\, x + P_y\, y = m$

$e_x + e_y = m \quad => \frac{e_x}{m} + \frac{e_y}{m} = 1$

share of expenditure $S_x + S_y = 1$

$P_x \frac{\partial x}{\partial m} + P_y \frac{\partial y}{\partial m} = \frac{\partial m}{\partial m} = 1$

$\frac{P_x X}{m} \cdot \frac{\partial x}{\partial m} \cdot \frac{m}{x} + \frac{P_y y}{m} \cdot \frac{\partial y}{\partial m} \cdot \frac{m}{y} = \frac{e_x}{m} \varepsilon_x^m + \frac{e_y}{m} \varepsilon_y^m = 1$

$S_x \varepsilon_x^m + S_y \varepsilon_y^m = 1$

$\varepsilon_x^m < 0$ inferior good => $\varepsilon_y^m > 0$

furthermore $\varepsilon_y^m > 1$ => Y is luxury.

$0 < \varepsilon_x^m < 1$ necessity => $\varepsilon_y^m > 1$ => Y is luxury.

∗ Homogeneous Function

Def：

$f(x, y)$ is homogeneous od degree k in X and Y

if $f(tx, ty) = t^k f(x, y)$ for all $t > 0$

n variable, $o<m<n$

Def：$f(x_1, x_2, \ldots, x_n)$ is homogeneous of degree k in $x_1, x_2, \ldots, x_m$

if $f(tx_1, tx_2, \ldots, tx_m, x_{m+1}, \ldots, x_n) = t^k f(x_1, x_2, \ldots, x_n)$

for all $t > 0$

Example 1:

$u(x, y) = \min\{\frac{x}{3}, \frac{y}{2}\}$

$u(tx, ty) = \min\{\frac{tx}{3}, \frac{ty}{2}\} = t \min\{\frac{x}{3}, \frac{y}{2}\}$

$= t\, u(x, y)$

$k = 1 => u(x, y) = \min\{\frac{x}{3}, \frac{y}{2}\}$ is homogeneous of degree 1 in X and Y

Example 2:

$u(x, y) = 24x + 6y$ # of boxes of Coke # of six−pack Coke

$u(tx, ty) = 24tx + 6ty = t(24x + 6y)$

$= t\, u(x, y)$

$k = 1 => u(x, y) = 24x + 6y$ is homogeneous of degree 1 in X and Y.

Example 3: Cobb-Douglas utility function

$u(x, y) = x^{0.2}y^{0.4}$

$u(tx, ty) = (tx)^{0.2}(ty)^{0.4} = t^{0.6}x^{0.2}y^{0.4}$

$= t^{0.6}\, u(x, y)$

$x^{0.2}y^{0.4}$ is homogeneous of degree 0.6 in X and Y.

Example 4: $u(x, y) = x^{0.5} + y$

$u(tx, ty) = (tx)^{0.5} + ty$

$= t^{0.5}\,(x^{0.5} + t^{0.5}y) \neq u(x, y)$

can't be arranged in $t^k(x^{0.5} + y)$ form

=> $u(x, y) = x^{0.5} + y$ isn't a homogeneous fct.

Example 5: Demand function

$x^* = x(P_x, P_y, m)$ ⟹ $x(tP_x, tP_y, tm) =?\ t^k\, x(P_x, P_y, m)$

$y^* = y(P_x, P_y, m)$ ⟹ $y(tP_x, tP_y, tm) =?\ t^k\, y(P_x, P_y, m)$

Are demand function homogeneous?

If yes, in which rariables?

(1) $max_{x,y}\, u(x, y)$

s.t. $P_x\, x + P_y\, y = m$

FOC => $MRS_{xy}\ (= \frac{Mu_x}{Mu_y}) = \frac{P_x}{P_y}$

$P_x\, x + P_y\, y = m$

=> $x^* = x(P_x, P_y, m)$

$y^* = y(P_x, P_y, m)$

(2) $\max_{x,y}\, u(x, y)$

s.t. $tP_x\, x + tP_y\, y = tm$

FOC => $MRS_{xy}\ (= \frac{Mu_x}{Mu_y}) = \frac{P_x}{P_y}$

$\not{t}P_x\, x + \not{t}P_y\, y = \not{t}m$ => $P_x\, x + P_y\, y = m$

⇨ $x^* = x(tP_x, tP_y, tm) = x(P_x, P_y, m)$

$y^* = y(tP_x, tP_y, tm) = y(P_x, P_y, m)$

solved in (2) / solved in (1)

$t^k = 1 \Rightarrow k = 0$

⇨ Demand function $x(P_x, P_y, m)$, $y(P_x, P_y, m)$ are homogeneous of degree 0 in P_x, P_y and m. Money illusion 貨幣幻覺

suppose $f(x, y)$ is homogeneous of degree k in X and Y

$f(tx, ty) = t^k f(x, y)$ for all $t>0$

$$\frac{df(tx,ty)}{dt} = \frac{dt^k f(x,y)}{dt}$$

$$f_x \frac{dtx}{dt} + f_y \frac{dty}{dt} = kt^{k-1} f(x,y)$$

$$f_x X + f_y Y = kt^{k-1} f(x,y)$$

$$Let\ t = 1, \quad f_x X + f_y Y = kf(x,y) \quad Euler\ Teorem\ Equation$$

An application of Euler Teorem

$$\left.\begin{aligned} x^* &= x(P_x, P_y, m) \\ y^* &= y(P_x, P_y, m) \end{aligned}\right\} \text{ are homogeneous of degree } k \text{ in } P_x, P_y \text{ and m}$$

According to Euler Teorem, we have

$$\frac{\partial x}{\partial P_x} \cdot P_x + \frac{\partial x}{\partial P_y} \cdot P_y + \frac{\partial x}{\partial m} \cdot \frac{m}{x} = kx = 0$$

$$x \neq 0, \ \frac{\partial x}{\partial P_x} \cdot \frac{P_x}{x} + \frac{\partial x}{\partial P_y} \cdot \frac{P_y}{y} + \frac{\partial x}{\partial m} \cdot \frac{m}{x} = 0$$

$$-\varepsilon_x^p + \varepsilon_{xy} + \varepsilon_x^m = 0$$

$$\left| \frac{\frac{\Delta x}{x}}{\frac{\Delta P_x}{P_x}} \right| = \frac{\Delta x}{\Delta P_x} \frac{P_x}{x}$$